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Prediction of Premixed Flame Temperature in a Radial Microcombustor

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ABSTRACT In this paper, an analytical procedure is applied to a typical radial microcombustor to obtain a relation between wall and flame temperature. In order to establish a correlation, heat transfers within the reaction zone and through the walls are investigated. Assuming a linear flame temperature profile along the flame zone, a relation between flame and inner wall temperature at the end of combustion is derived. Then by considering wall thermal resistance, the flame temperature is related to the outer wall temperature. In this study, the flame flow is assumed to be laminar and *Nu* number along the radius is determined by a polynomial function. An available experimental investigation is used to validate the proposed relation through comparing measured flame radius by the flame radius which is calculated by predicted flame temperature and a good agreement was achieved.

KEYWORDS Microcombustion, Premixed flame, Flame Temperature

I. INTRODUCTION

The advent of micro electromechanical systems (MEMS) is raising a strong demand for small power sources. Due to the fact that micro- and meso-scale hydrocarbon fuelled combustors have about 100 times higher energy intensity as compared with the traditional electromechanical batteries, extensive efforts are being made in order to improve small scale combustion defects including the increased heat loss due to large surface area-to-volume ratio [1,2] and the wall radical quenching [3]. Many studies were carried out aiming to promote some fundamental issues such as flammability, flame stability, flame speed and wall temperature. To serve this aim, various geometrical shapes of microcombustors were investigated such as cylindrical tubes or rectangular channels or different gaseous fuels including H_2 and CH_4 were investigated [4]. Maruta et al. [5, 6] studies the combustion of premixed CH_4-air mixture in an externally heated quartz tube. They concluded that a stable premixed flame can exist at high or low mixture velocities. Afterward, Kumar et.al [7,8] performed an experimental study on flame dynamics of a heated radial microcombustor utilizing premixed CH_4-air as fuel. This geometry can be employed in micro power generation devices, for example disk-type micro gas turbines [9]. In addition to the conventional stable flame, some other unstable flame regimes like rotating pelton-like flame (the flame revolves around the combustor as a pelton wheel), traveling flame and so on were observed [7,8]. The investigated variables in Kumar et.al experiments include microchannel gaps, fuel-air equivalence ratio and inlet mixture velocities. In accordance with these studies, the flame regime diagrams were illustrated to discrete different regions of characterized flame patterns. Numerical and analytical models for prediction of various parameters in micro combustors provided an extensive contribution to the field. Raimondeau et al. [10] simulated the CH_4–air flame propagation in micro tubes and claimed that the stability of the flame can be obtained under specific conditions such as preheating of the mixture and wall insulation. Norton and Vlachos [11, 12] conducted a study on the combustion of CH_4–air and C_3H_8–air mixtures within two parallel plates and their results showed that there is a strong thermal coupling between the gases and the combustor wall. Typically, in all of the combustors, a parameter which characterizes the existence of combustion is the flame temperature. It is also considered as an important factor to study flame structure in micro combustors. Besides, flame temperature is significant since by which flame speed and flame thickness can be determined [13, 14]. However, according to the small size of micro-combustors, it is usually hard to fit and utilize the thermocouples to measure flame temperature. For example, the existence of a thermocouple in the flame region of a micro-combustors can virtually alters the whole flame pattern. To suppress the effect, the fine wire thermocouples may be used but these are probable to break shortly after being exposed to the flame [15]. On the other hand, the outer wall temperature of the combustor is in comparison easy to record. If it is possible to establish a correlation between flame and wall temperature then the difficulties can be overcome.

This paper plans to extend the Li. et.al study [15] to establish a relation between flame and wall temperature of a typical radial micro-combustor. Two separate control volumes within the combustor were investigated to derive a relation by which flame temperature is related to outer wall temperature. In order to validate the model, the flame thickness is calculated based on the flame temperature predicted by the proposed relation. Then the flame thickness is compared with the experimental investigation results of Aiwu Fan et.al [17] on a radial micro-channel.

II. FLAME AND WALL TEMPERATURE DIFFERENCE

We consider a premixed flame flow in a radial microchannel of radius L. The inlet mixture is introduced to the combustor through a central passage. The fuel delivery tube diameter is d and the gap between parallel disks is S. The sketch of the investigated combustor is depicted in Fig.1.

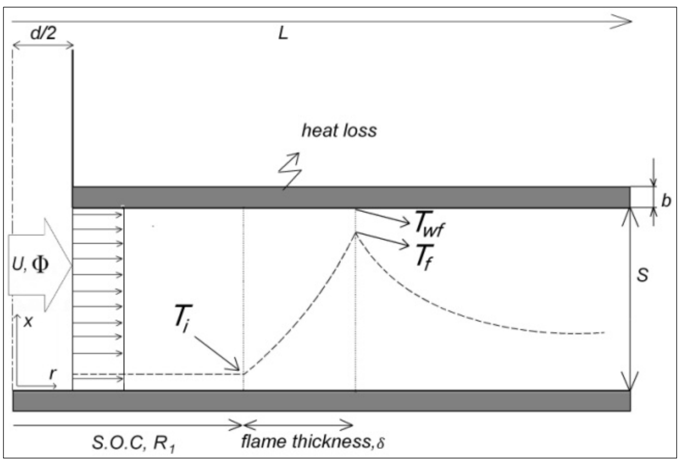

Figure 1. 2-D sketch of investigated radial microcombustor

For the sake of simplicity, we assume that the narrow channel considered here is in the range in which the flow is laminar. In addition, the temperature difference between the flame and the inner wall is supposed to increases linearly. This seems to be true according to the numerical simulation available in literature. For example the study of Li. et.al supports this statement [15]. The governing equations describing energy balance are fitted in 1-D cylindrical coordinates *(x, r, φ)* which the radius of the combustor lies along *r* axis and positive in flow direction. We choose $r=0$ at the center line of the disks and $x=0$ at the bottom of the combustor. The following assumptions are applied to the flow and combustion of the radial combustor: 1) The flow is uniform across the φ direction; 2) Dufour effect is neglected; 3) no gas radiation; 4) no viscous dissipation; 5) inert wall (no absorption/desorption of species); and; 6) steady state. Before trying to relate flame and wall temperature of the combustor, it is essential to approve the fact that temperature difference between flame and inner wall changes linearly across the flame region because based on this fact, the relation between flame and wall temperature will be formed. This fact is extensively discussed in Li. et.al [15] study by numerical simulation of a micro-combustor using momentum and energy governing equations. According to the similarity of radial and axial governing equations in cylindrical coordinates, the linear temperature difference between wall and flame can be dealt for a radial microcombustor, to.

III. RATIO OF RADIAL TO AXIAL CONDUCTION

To achieve a relationship between the flame temperature and the wall temperature, a scale analysis is used to relate the axial and the radial heat conduction in microcombustor. We choose $\Delta T=T_f-T_i$ as the flame temperature scale. Assuming the gas mixture as an ideal gas, the gas enthalpy is designated as $h=c_pT$, where c_p is the average specific heat capacity of the gas mixture. In addition, we choose the unburnt gas density, ρ_0, as the density scale; the disks gap S as the axial-length scale; the flame thickness δ as the radial-length scale, where $\delta=\alpha/S_L$, and $\alpha=k/\rho C_p$, the thermal diffusivity, and the flame speed S_L as the scale of the radial velocity. Given the previous scales, it is appropriate to set the scale domain as $\delta \times S$. Within this domain, we have $v \sim S_L$, $r \sim \delta$, $x \sim S$, $T \sim \Delta T$, and $\rho \sim \rho_0$.

The energy equation in cylindrical coordinates may be declared as:

$$\frac{\partial(\rho u h)}{\partial x}+\frac{1}{r}\frac{\partial(\rho v h r)}{\partial r}$$
$$=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}+r\right)+\dot{q} \qquad (1)$$

By applying the scale analysis terms on Eq.1 we can express

$$\frac{u\Delta T}{S} \quad + \quad \frac{S_L \Delta T}{\delta} \quad = \quad \frac{\propto \Delta T}{S^2}$$

Axial convection Radial convection Axial conduction

$$+ \quad \frac{\propto \Delta T}{\delta^2} \quad + \quad \frac{\dot{q}}{\rho_0 C_P}$$

Radial conduction Volumetric heat generation

Dividing by $\frac{\propto \Delta T}{S^2}$ gives:

$$\frac{uS}{\propto} + \frac{S_L S^2}{\propto \delta} = 1 + (\frac{S}{\delta})^2 + \frac{\dot{q} S^2}{\rho_0 C_p \alpha \Delta T} \qquad (2)$$

This implies that:

$$\frac{Q_{radial}}{Q_{axial}} = \frac{\dot{q}}{\dot{i}} = \left(\frac{S}{\delta}\right)^2 \qquad (3)$$

It should also be noted that since combustor walls are considered thin, the heat conduction along the walls are negligible. Consequently the heat recirculation effect which seems to be important in thick-wall combustors [4] is ignored.

IV. FLAME AND INNER WALL TEMPERATURE

The previous part showed that the ratio of the radial to the axial heat conduction per unit volume is $(\frac{S}{\delta})^2$. This conclusion will be used to make a relation between flame and inner wall temperature. One important assumption to make is that the flame travels almost with the same speed as inlet mixture velocity. Therefore, a flat flame front across the combustor is expected. According to the small scale of the combustor gap (*b*), the flame temperature is assumed to be invariant in the axial direction which means *T* only changes with *r*. It is important to note that since *Nu* number is a function of combustor radius, in this paper the combustor radius, *r*, is normalized and will be written as $r^*=r/L$ to keep the *Nu* number as a dimensionless number. Within a finite distance *dr* in the radial direction, the flame temperature is T_{r^*}, the inner wall surface temperature is $T_{w_{r^*}}$, and the temperature difference $T_{r^*} - T_{w_{r^*}}$,is proportional to $r - R_1$ (the radial length of the flame from S.O.C). This assumption is made based on the results of the numerical simulations which are reported by Li. et.al and described in Sec. II. Considering $r = r^* L$ gives

$$T_{r^*} - T_{w_{r^*}} = \frac{(r^* L - R_1)}{\delta}(T_f - T_{wf}) \qquad (4)$$

Where T_{wf} is the inner wall temperature and T_f is the flame temperature at the end of combustion. Also, it is assumed that the flame temperature T_{r^*} increases linearly with r along the flame zone [15] therefore, we have

$$\frac{T_{r^*} - T_i}{T_f - T_i} = \frac{r^* L - R_1}{\delta} \tag{5}$$

Where $r^*L - R_1$ again represents the radial length of the flame from S.O.C .
According to the previous considerations, we derive the radial heat conduction rate per unit volume The axial heat conduction rate, $d\dot{L}$, from the flame to the wall within the control volume of length dr may be written as:

$$d\dot{L} = 2 \times \left[2\pi r^* L \left(\frac{k}{2s}\right) Nu_{r^*}.\left(T_{r^*} - T_{w_{r^*}}\right)\right] dr^* \tag{6}$$

In the above differential equation, $Nu_{r^*} = h_{r^*} D_h/k$ where h_{r^*} is the local heat convection coefficient, D_h is hydraulic diameter of the combustion chamber and is $2S$ and k is the heat conduction coefficient of the mixture at the mean temperature. Besides, since the heat is dissipated through two parallel walls, the axial heat conduction is doubled. In order to describe the variations of Nu along the radial direction, a polynomial function, which extracted from study of Esfahani et.al, is employed [16]. In their work, the laminar flow field between parallel disks is investigated numerically. The designated function to characterize Nu variations is as followed:

$$Nu_{r^*} = -22.5\, r^{*3} + 55.4\, r^{*2} - 45.9\, r^* + 14.5 \tag{7}$$

Replacing temperature difference from eq.4:

$$d\dot{L} = 2\times \left[2\pi r^* L \left(\frac{k}{2s}\right) Nu_{r^*}.\frac{(r^*L - R_1)}{\delta}\left(T_f - T_{wf}\right)\right] dr^* \tag{8}$$

Integrating $\dot{l}$ from $r^* = R_1/L$ to $r^* = (R_1+\delta)/L$ and dividing by the volume $\pi\left[(R_1+\delta)^2 - {R_1}^2\right].S$ leads to the following relation:

$$\dot{l} = 2Lk.RD.\delta\left(T_f - T_{wf}\right) / \, S^2\left[(R_1+\delta)^2 - {R_1}^2\right] \tag{9}$$

Where RD represents the integration of $Nu_{r^*}.\frac{(r^*L - R_1)}{\delta}$ over the flame region.
On the other side, we assume conductive heat transfer along r as heat transfer in a flat plate we write:

$$\dot{Q} = \frac{1.5\times 2\pi R_1 S k\left(T_f - T_i\right)}{\delta} \tag{10}$$

Where 1.5 approximates heat conduction for flat plate. Dividing by the volume becomes:

$$\dot{q} = 3R_1 k \frac{\left(T_f - T_i\right)}{\left[(R_1+\delta)^2 - {R_1}^2\right]\delta} \tag{11}$$

From the earlier discussion, the ratio of the radial to the axial heat conduction per unit volume is:

$$\frac{\dot{q}}{\dot{l}} = \left(\frac{S}{\delta}\right)^2$$

By combining equations 11, 9 and 3 the following wall relation is derived:

$$T_{wf} = T_f - 1.5R_1\left(T_f - T_i\right)/L.RD \qquad (12)$$

Evidently, flame thickness (δ) and combustor gap (S) are disappeared from the equation which implies the fact that the equation can be used for various combustor gaps.
Considering the micro combustor sketch in Fig.1 and assuming the constant heat conductivity of the combustor walls, k_w, the heat flux from the flame to the wall at $r=\delta$ can be written as:

$$\dot{q}_{in} = h_\delta\left(T_f - T_{wf}\right) \qquad (13)$$

Where T_{wf} is the inner wall temperature. The dissipated heat flux through the walls at $r=\delta$ can be expressed as

$$\dot{q}_{out} = \frac{k_w(T_{wf} - T_{wo})}{b} \qquad (14)$$

where b is wall thickness. With the assumption that there is no recirculated heat conduction in the walls (walls are thin) gives

$$\dot{q}_{in} = \dot{q}_{out} \qquad (15)$$

Replacing T_{wf} of equation 12 in the equations 13, 14 and 15 provides:

$$T_f = \frac{\frac{L.RD}{1.5R_1}T_{wo} - (1 + Bi)T_i}{\left(\frac{L.\overline{Nu}}{1.5R_1} - Bi - 1\right)} \qquad (16)$$

In this equation $Bi = \frac{h_\delta b}{k_w}$ and $h_\delta = kNu_\delta/D_h$ and it is clear that in order to obtain the correct Nu number, the flame thickness which it's self is a function of flame temperature should be determined first. To serve this aim, an iterating process based on trial and error is employed which can simultaneously calculate the flame temperature and flame thickness from a specified wall temperature.

V. MODEL VALIDATION AND DISCUSSION

For the purpose of model validation, the proposed relation in this paper (Eq.16) is used to obtain the radial combustor flame thickness. Then by adding the flame thickness to the radius of start of combustion (S.O.C) which is located by analysing the photographs of a typical radial micro combustor, the overall flame radius is determined and compared to the experimental results.

The schematic figure of the microcombustor by which the model is validated is shown in Figure 2. Two circular quartz plates (Ø50) were maintained parallel to each other within an accuracy of ±0.1◦. The bottom plate is externally heated to provide a positive temperature gradient. Consequently, the top plate is heated by the bottom plate via heat convection and radiation. The methane-air mixture is introduced to the center of the combustor by 4m/s velocity at 300^{o}K and the combustion products leave from combustor rim [17].

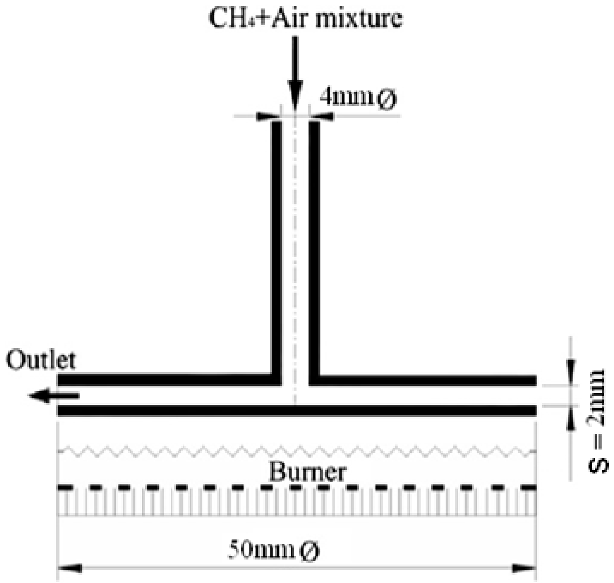

Figure 1. Schematic figure of experimentally investigated radial microcombustor

As it was mentioned earlier, the predicted flame temperature is used to predict the flame thickness. According to the thermal flame theory [14], the flame thickness, δ, is related to S_u (non-adiabatic flame speed) by

$$\delta = \frac{k_f}{\rho_f C_{p,f} S_u} \qquad (17)$$

Where $k_f, \rho_f, C_{p,f}$ are defined at flame average temperature. The non-adiabatic flame speed can be correlated to the flame temperature as follows [13]:

$$S_u = S_u^a . e^{E_a/2R_u(\frac{1}{T_f^a}-\frac{1}{T_f})} \qquad (18)$$

In the above equation S_u^a is adiabatic flame speed, E_a is the activation energy for the reaction, R_u is universal gas constant, T_f^a is the adiabatic flame temperature and T_f is the flame temperature. Finding the location of S.O.C is done by comparing the photographs of the same radial combustor which are provided by Kumar et.al [8].

To find flame temperature and subsequently flame thickness equations 16 to 18 are employed. First, an assumed outer wall temperature is introduced to Eq.16 and with an initial Nu number, the flame temperature is calculated. Then the flame temperature is used to calculate flame thickness by Eqs. 17 and 18. By determining the flame thickness, $\overline{Nu}$ number in Eq.16 which depends on the flame thickness is corrected and the flame temperature is calculated again. The process continuous until the flame temperature is converged. The flame thickness which is determined on the basis of final flame temperature will be compared with experimental results of Fan et.al study [17]. For this case, in eq.16 the inlet velocity is V_i=4m/s, inlet temperature is T_i =300°K, RD=1.5×10^5, b=1mm, k_w=9W/m.K and R_1 is 6mm.

Accordingly the predicted and measured flame radius is provided in fig.4 which shows a good agreement.

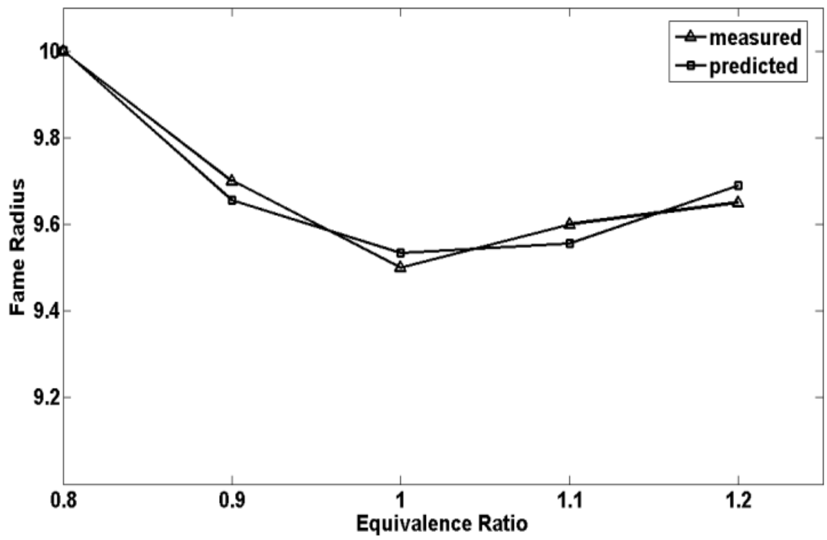

Figure 2. Comparison between measured and predicted flame radius

VI. EFFECT OF COMBUSTOR WALL CHARACTERISTICS

According to the described model validation in Sec.4, we can now develop the model in order to investigate the effect of other combustor parameters such as wall characteristics on the flame and wall temperature. For this purpose assuming the unchanged flame thickness, we examined different wall *Biot* numbers on the flame and wall temperatures for different equivalence ratios. Changing wall *Bi* can be accomplished by changing the wall thickness or wall material. As it is clear in Table.1, increasing *Bi* can increase both flame and wall temperature, which drastically can avoid flame quenching that is a challenge in forming a stable flame region. This is due to the fact that by increasing Bi number, the thermal resistance of walls is promoted.

	$Bi = 0.01$		$Bi = 0.05$		$Bi = 0.1$	
φ	T_{flame} (K)	T_{wall} (K)	T_{flame} (K)	T_{wall} (K)	T_{flame} (K)	T_{wall} (K)
0.8	1802	1418	1906	1324	1980	1275
0.9	1917	1493	2065	1433	2148	1377
1	2011	1587	2143	1517	2250	1485
1.1	1993	1526	2118	1492	2214	1438
1.2	1902	1489	2054	1421	2180	1369

Table1. Computed flame and wall temperatures with various wall Biot numbers for different equivalence ratios output

VI. CONCLUSION

An analytical model based on a scale analysis is developed in order to establish a relation between wall and flame temperature of a radial microcombustor which is fed by premixed methane-air. Since the measurement of flame temperature in micro scale combustors experimentally is quite hard, the proposed relation in this paper can be helpful. With this equation, the flame temperature can be determined by measuring the outer wall temperature of a radial combustor which is relatively easier to obtain. The model is validated by available data from an experimental investigation on a typical radial micro-combustor. First, the flame temperatures were obtained by the proposed model in this paper. Then, the flame thickness was calculated based on the flame temperature. Finally, the predicted flame thicknesses were compared with flame thicknesses reported by experimental tests. The model is employed in order to investigate the effects of wall Biot numbers on flame and wall temperature and it was revealed that a minor increase in Bi can lead to significant increase in flame and wall temperatures.

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